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CONTROL THEORY APPROACHES TO OPTIMIZE INFORMATION FLOW IN THE FACE OF UNCERTAINTY AND CONFLICTING DATA

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Abstract. The rapid spread of inaccurate information through social media has become a major concern in recent years. This paper presents a mathematical model for analyzing the impact of inaccurate information on the spread of accurate information in a network. We aim to evaluate the effectiveness of three different strategies of control: media literacy programs and fact-checking and a combination of both. The model is analyzed using the maximum principle of Pontryagin to derive the optimal control strategies for minimizing the spread of inaccurate information while maximizing the spread of accurate information. A numerical simulation is presented to illustrate the effectiveness of the control strategies, and statistical analysis is performed to compare the impact of the different control strategies on the spread of information. The results demonstrate that the combination of media literacy programs and fact-checking is the most effective strategy for increasing the spread of accurate information and reducing the spread of inaccurate information. These findings have important implications for the design of effective strategies to combat the spread of misinformation and promote the spread of accurate information in a network.

Keywords: Information control, optimal control, information spread, network analysis, statistical analysis, misinformation.

AMS Subject Classification: 94A05,93C10, 93C95.

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1 Introduction

Rumors can have a significant impact on individuals, groups, and society as a whole. They can spread quickly and affect people's opinions, decisions, and actions. In some cases, rumors can cause harm, such as damaging a person's reputation, creating false beliefs, or inciting fear and panic. In other cases, rumors can serve as a source of entertainment or provide a sense of community. However, it's important to be critical of the information presented in rumors and verify them with reliable sources before accepting or sharing them (DiFonzo & Bordia, 2007; Jansen et al., 2009; Chua & Banerjee,, 2018).

Rumors can also impact the economy and the stock market, as well as trigger political and

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social movements. For example, false rumors about a company's financial performance can cause its stock price to plummet, while rumors about a political leader can influence public opinion and shape election outcomes.

Moreover, rumors can contribute to the spread of misinformation and harm public health efforts, as seen in the case of misinformation surrounding COVID-19. In such cases, false rumors can lead to the delay or non-adoption of recommended health practices, putting people at greater risk of harm.

In conclusion, while rumors can be spread quickly and easily, it's important to be aware of their potential impact and to carefully consider the source and credibility of the information before accepting or sharing it (Ahmed et al., 2020; Brennen et al., 2020).

Rumors can also lead to further consequences beyond just the initial impact. For instance, rumors about a person's character or behavior can lead to discrimination or ostracism from their community. In some extreme cases, rumors can lead to violence or even armed conflict, as rumors can be used to stir up emotions, create tensions, and mobilize people. Furthermore, rumors can have a lasting impact on people's lives, even after they have been proven to be false. This is particularly true in the age of the internet, where false information can spread rapidly and be archived indefinitely (Jansen et al., 2009).

In the workplace, rumors can cause low morale and decreased productivity, as employees may become distracted, distrustful, or concerned about the future of their jobs. Companies can also suffer damage to their reputation if rumors about their business practices or products are not effectively managed. Therefore, it is important to understand that rumors can have far-reaching and long-lasting effects and should be treated with caution. Encouraging critical thinking and media literacy, promoting accurate and transparent information, and addressing rumors in a responsible and timely manner can help to mitigate their negative impacts (Carroll & McCombs, 2003; Austin et al., 1990).

During the COVID-19 pandemic, there were many rumors and false information circulating about the virus, including its origins, symptoms, and treatments. Health organizations and governments worked to counteract these rumors by providing accurate information and countering them with trustworthy rumors. For example, they released information about the symptoms of the virus and how to prevent its spread, which helped to dispel the rumors and reduce their impact (Lhous et al., 2020; Zakary et al., 2020).

Also companies often face rumors about the safety of their products. In such cases, they may release statements or conduct studies to show that their products are safe, which can help to mitigate the impact of the rumors. For example, a food company may release information about the rigorous safety tests their products undergo, which helps to dispel rumors about their safety and quality.

Rumors often circulate about candidates and their policies. Campaign teams can counteract these rumors by releasing information about the candidate's position and their record, which helps to dispel the rumors and reduce their impact (Bidah et al., 2020a,b). For example, a political campaign may release information about a candidate's voting record or their position on a particular issue, which helps to dispel rumors and provide context to the public. Publicists and representatives may release statements or conduct interviews to dispel the rumors and provide accurate information. For example, a celebrity may conduct an interview to deny a rumor about their personal life or to set the record straight, which helps to mitigate its impact.

There have been numerous rumors and misinformation campaigns related to the safety and effectiveness of vaccines, leading some people to question the value of vaccines and to refuse to vaccinate themselves or their children. This has had a significant impact on public opinion about the importance of vaccines in protecting public health. Furthermore, rumors and conspiracy theories about the science behind climate change, suggesting that the evidence for global warming is exaggerated or even manufactured. These rumors have influenced public opinion about the reality of climate change and the urgency of taking action to address it (Larson et al., 2014; Betsch et al., 2010; Lewandowsky et al., 2013; Van der Linden et al., 2017).

Political scandals often generate rumors and speculation about the motives and behavior of politicians. These rumors can have a significant impact on public opinion about the credibility and integrity of political leaders, and can influence election outcomes (Pomper, 2001; Vaccari et al., 2023).

These examples show how rumors can shape public opinion and lead to a skewed perception of reality. It is important to be critical of information and to seek out credible sources in order to form accurate opinions.

Social media platforms can be a powerful tool for spreading rumors, but they can also be used to counteract rumors and provide accurate information. For example, in response to a rumor about a public figure, a trusted source, such as a close friend or family member, may use social media to dispel the rumor and provide accurate information. This can help to reduce the impact of the rumor and provide a more balanced view of the situation. Thus, social media can be both a source of rumors and a tool for mitigating their impact.

People can mitigate the impact of a rumor by verifying the information, that is before spreading any information, it's important to verify its accuracy. This can be done by checking reliable sources or asking the person or organization directly involved. Then, it's important to find accurate and credible alternative information that can be spread to counteract the original rumor. This can be done through personal networks, social media, or other forms of communication. Furthermore, it's important to monitor the situation and respond to any further developments as they occur. This may involve updating the alternative information and spreading it again.

By spreading accurate and trustworthy information, people can help to counteract the negative impact of a rumor and reduce its spread. However, it's important to be mindful of the potential consequences of spreading false information, as it can further contribute to the spread of misinformation.

Additionally, one can also use other strategies to mitigate the impact of rumors:

- If the rumor is about a specific person, group, or organization, it can be helpful to address it directly. This can be done by releasing a statement or holding a press conference to provide accurate information and set the record straight.

- Rumors often gain traction because they tap into people's fears and emotions. By providing context and additional information, people can help to dispel the rumors and reduce their impact.

- Encouraging people to think critically and question the sources of information they receive can help to mitigate the impact of rumors. This can be done by promoting media literacy and critical thinking skills.

- Partnering with trusted organizations, such as media outlets, government agencies, or non-profit organizations, can help to counteract rumors and provide accurate information to the public.

Another strategy to mitigate the impact of rumors is to use humor or satire. Humor can be a powerful tool in dispelling rumors, as it can make the situation less tense and diffuse the emotions behind the rumor. By using humor, people can challenge the rumor in a way that is not threatening or confrontational. Moreover, involving influential individuals or organizations in the rumor control efforts can also be effective. These individuals or organizations can use their platform and credibility to spread accurate information and counteract rumors.

It's also important to note that rumors can take different forms, such as conspiracy theories, hoaxes, and false information, so the approach to mitigating their impact may vary. For example, conspiracy theories often stem from a lack of trust in institutions and authorities, so addressing the root causes of that distrust can be more effective in mitigating their impact than simply debunking the conspiracy theory itself. Finally, it's crucial to monitor the situation and respond to any new developments. Rumors can evolve and change over time, so it's important to be vigilant and continue to spread accurate information as new information becomes available.

Mitigating the impact of rumors requires a combination of accurate information, critical thinking, and effective communication. It's important to remember that mitigating the impact of rumors takes time and effort, and it's not always possible to completely eliminate their impact. However, by taking proactive steps and working together, people can help to minimize the damage caused by rumors and promote accurate information. By using a combination of these strategies, people can help to reduce the harm caused by rumors and promote accuracy in public discourse.

Mathematical models often used to predict the spread of rumors and to understand how rumors spread in different contexts and under different conditions. Furthermore, to analyze the structure of social networks and to understand the role of different actors in the spread of rumors. Mathematical models used to support decision making by providing insights into the likely impact of different rumors mitigation strategies. For example, to simulate the spread of rumors in response to different countermeasures, or to identify the early signs of a rumor outbreak and to trigger early mitigation efforts before the rumor becomes widespread. This information can be used to identify the sources of rumors and to develop strategies for mitigating their impact. In the literature, several mathematical models are developed to measure the impact of rumors on individuals, organizations, and society as a whole (Rachik et al., 2020a, 2021, 2020b; Austin et al., 2020).

Another application of mathematical models in mitigating rumors is in the area of sentiment analysis. Sentiment analysis uses natural language processing techniques and machine learning algorithms to analyze the tone and sentiment expressed in text data, such as social media posts.

By using sentiment analysis to monitor the spread of rumors on social media, organizations can detect rumors early on and respond quickly to counteract their impact. For example, sentiment analysis used to identify negative sentiment associated with a particular product or brand, which may indicate the spread of a damaging rumor. In response, the organization can release accurate information or conduct a targeted public relations campaign to mitigate the impact of the rumor (Hu & Liu, 2004; Alrefai et al., 2018; Liu, 2012).

Network analysis uses graph theory and other mathematical tools to understand the structure of social networks and to identify the key actors and relationships that drive the spread of rumors. By using network analysis to understand the spread of rumors, organizations can identify the sources of rumors and target their mitigation efforts more effectively. For example, one use network analysis to identify the most influential individuals in a social network and to develop targeted campaigns to correct misinformation and dispel rumors. In doing so, organizations can develop algorithms to automatically detect rumors and to verify the accuracy of information in real-time.

In the area of risk assessment, mathematical models used to estimate the risk associated with the spread of rumors and to prioritize the deployment of mitigation efforts. For example, a model may be used to estimate the likelihood that a rumor will spread rapidly in a particular community or online network. This information can be used to develop targeted interventions to reduce the spread of the rumor and mitigate its impact. Similarly, a model may be used to estimate the impact of a rumor on public opinion, public health, or the economy, which can help to prioritize the deployment of resources and efforts to counteract the rumor.

In (Zubiaga et al., 2018), authors presented a survey of the methods and techniques used to detect and resolve rumors in social media, including the use of mathematical models to describe the dynamics of rumor propagation and to develop strategies for controlling the spread of rumors.

Other studies (Xiao et al., 2019; Jain et al., 2020; Zhang et al., 2016) demonstrate the potential of mathematical models to support decision-making in risk assessment and to develop effective strategies for mitigating the spread of rumors and misinformation.

In conclusion, mathematical models can play a key role in mitigating the impact of rumors by providing valuable insights into the spread and impact of rumors, and by supporting the development of effective mitigation strategies. Whether used for sentiment analysis, network analysis, risk assessment, or fact-checking, mathematical models have the potential to help organizations reduce the harm caused by rumors and promote accuracy in public discourse.

2 The mathematical model

The spread of information, whether true or false, can have a profound impact on a society. With the rise of social media and other forms of digital communication, information can spread quickly and easily, often without proper vetting or fact-checking. As a result, it is essential to understand how information spreads through a population and the factors that influence its direction and speed. In this paper, we present a mathematical model designed to study the impact of information on other information. Our model can be used to investigate the dynamics of how information spreads and how it can be influenced or controlled. By analyzing the factors that contribute to information flow, we can develop strategies to mitigate the spread of rumors and misinformation and promote the dissemination of accurate information. Our research can help inform policymakers, educators, and the public on the most effective ways to combat the spread of misinformation, ultimately leading to a more informed and knowledgeable society. Through this modeling approach, we can better understand the dynamics of information flow and work towards a more trustworthy and reliable information environment.

The mathematical model describing the interaction of two pieces of information that can evolve and change over time is as follows:

$$S' = -\beta_1 S I_1 - \beta_2 S I_2 \tag{1}$$
$$I_1' = \beta_1 S I_1 - r_1 I_1 + (p_1 - p_2) I_1 I_2 + \rho_1 I_1 R_2 \tag{2}$$
$$I_2' = \beta_2 S I_2 - r_2 I_2 + (p_2 - p_1) I_1 I_2 + \rho_2 I_2 R_1 \tag{3}$$
$$R_1' = r_1 I_1 - \rho_2 I_2 R_1 \tag{4}$$
$$R_2' = r_2 I_2 - \rho_1 I_1 R_2 \tag{5}$$

Where $S(0) \geq 0$, $I_1(0) \geq 0$, $I_2(0) \geq 0$, $R_1(0) \geq 0$, and $R_2(0) \geq 0$.

This model describes how the spread of one piece of information can influence the spread of another piece of information, and how the interaction between the two can result in the suppression or amplification of either or both informations. Where S is the number of the indifferent people, while the I_k $(k \in \{1; 2\})$ refers to the number of individuals who have access to information k and are capable of transmitting it to others. Some of these individuals may be persuaded by the information k and actively promote it to others, while others may not necessarily believe the information themselves. Nonetheless, both types of individuals play a significant role in the spread and impact of the information, as they can generate support and enhance its perceived credibility.

People can change their minds about information if they are exposed to evidence, facts, or arguments that challenge or reinforce their initial beliefs. Social influences, personal experiences, cognitive biases, and emotional factors can also play a role in shaping people's attitudes towards information. Additionally, exposure to multiple sources of information can increase people's critical thinking skills, help them assess the credibility of sources, and make informed decisions about what to believe.

Indifferent persons S can change their attitude and become advocates of information k if they are exposed to and convinced by persuasive arguments, credible sources, or personal experiences that support this information. Social influence from friends, family, or peers who are already convinced by information k can also play a role in changing the indifferent person's attitude.

Table 1: Parameters description

Parameter ($k \in \{1;2\}$)	Description ($k \in \{1;2\}$)
β_k	The contact rate between an indifferent S and I_k
p_1	The polarization rate from I_2 to I_1
p_2	The polarization rate from I_1 to I_2
r_k	Loss of interest rate of I_k

Additionally, emotional responses, personal biases, and past experiences can also play a role in convincing S to adopt information k at a rate β_k ($k \in \{1;2\}$).

Advocates of information 1 (I_1) can change their mind and become advocates of information 2 at a polarization rate p_2 if they are exposed to new and credible evidence that contradicts or provides a different perspective on their previous beliefs. This can also happen if they have a personal experience or encounter someone (I_2) who challenges their beliefs and presents a compelling argument for the new information (Information 2). Additionally, social pressure and group dynamics can also play a role in changing one's mind, as people may adopt beliefs that are more in line with those of their peers and social networks.

I_k can lose interest and become indifferent again at a rate r_k due to various reasons such as access to counter information, dis-confirming evidence, changes in personal beliefs and values, or a shift in the social context in which the information is being discussed. It's also possible for an individual's level of involvement and engagement to decrease over time, causing them to lose interest in the information. Additionally, people may become exposed to alternative viewpoints or narratives that challenge their current beliefs, leading them to re-evaluate their stance on the information.

All the positive constants are described in the Table 1.

3 Optimal control problem

3.1 Presentation of the model with controls

In the context of mitigating the spread of misinformation, two commonly used control variables are fact-checking (represented by v) and media literacy programs (represented by u).

The control v represents the fact-checking control and which is used to verify the accuracy and credibility of information. It can be carried out by independent third-party organizations, journalists, or individuals who have the knowledge and expertise to evaluate the evidence and sources behind a claim. The goal of fact-checking is to identify and correct false or misleading information, and to provide a more accurate and reliable representation of the facts. This can be achieved through a variety of methods such as cross-referencing sources, interviewing experts, or conducting research

On the other hand, the control variable u is used to represent the effect of media literacy programs on the spread of information. These can be a powerful tool for combating the spread of misinformation and promoting the dissemination of accurate information. This control typically aims to educate the public on how to critically evaluate media content and identify misleading or false information.

The control variables are incorporated into the mathematical model to simulate the spread of information and evaluate the impact of interventions. By analyzing the effects of these controls, optimal strategies can be developed to mitigate the spread of misinformation (Information 2 in this case). For example, the effectiveness of fact-checking can be influenced by the trustworthiness of fact-checkers and the audience's willingness to accept corrected information. Similarly, the effectiveness of media literacy programs can be influenced by factors such as the level of engagement and access to reliable information sources.

It is important to note that these strategies are not mutually exclusive and can be used in combination to mitigate the spread of misinformation and rumors. Additionally, the effectiveness of these strategies can vary depending on the context and the specific scenario, and further research is needed to understand their impact in different situations.

Therefore, the controlled model takes the following form

$$S' = -\beta_1 S I_1 - \beta_2 (1-u) S I_2 \quad (6)$$
$$I_1' = \beta_1 S I_1 - r_1 I_1 + (p_1 - p_2) I_1 I_2 + \rho_1 I_1 R_2 \quad (7)$$
$$I_2' = \beta_2 (1-u) S I_2 - r_2 I_2 + (p_2 - p_1) I_1 I_2 + \rho_2 (1-v) I_2 R_1 \quad (8)$$
$$R_1' = r_1 I_1 - \rho_2 (1-v) I_2 R_1 \quad (9)$$
$$R_2' = r_2 I_2 - \rho_1 I_1 R_2 \quad (10)$$

Where $S(0) \geq 0$, $I_1(0) \geq 0$, $I_2(0) \geq 0$, $R_1(0) \geq 0$, and $R_2(0) \geq 0$.

3.2 Optimal control problem

Now, we consider an optimal control problem to minimize the objective functional

$$J(u,v) = \int_0^{t_f} \left(c_2 I_2(t) - c_1 I_1(t) + \frac{K_1}{2} u^2(t) + \frac{K_2}{2} v^2(t) \right) dt$$

where c_1 and c_2 are small positive constants to keep a balance in the size of $I_1(t)$ and $I_2(t)$, respectively. The positive constants K_1 and K_2 balance the size of quadratic control terms. The reason behind considering a finite time horizon is that the control period is usually restricted to a limited time window. The objective of our work is to minimize the number of people supporting the second piece of information by using possible minimal costs of applying control variables $u(t)$ and $v(t)$ attempting to increase the number of advocates of information 1.

We seek an optimal control pair (u^*, v^*) such that

$$J(u^*, v^*) = min \{ J(u,v) | \ (u,v) \in U \} \quad (11)$$

subject to (6)-(10). Where

$$U = \{(u,v) | u, v \text{ measurables}, 0 \leq u(t) \leq 1, \ 0 \leq v(t) \leq 1, \ t \in [0, t_f]\} \quad (12)$$

In order to find an optimal solution, first we define the Lagrangian and the Hamiltonian for our optimal control problem. In fact, the Lagrangian of the optimal problem is given by

$$\mathcal{L}(I_1, I_2, u, v) = c_2 I_2(t) - c_1 I_1(t) + \frac{K_1}{2} u^2(t) + \frac{K_2}{2} v^2(t)$$

3.3 Characterization of the optimal controls

We seek the minimal value of the Lagrangian. To accomplish this, we define the Hamiltonian $\mathcal{H}$ as follows

$$\begin{aligned} \mathcal{H} &= \mathcal{L}(I_1, I_2, u, v) \\ &+ \lambda_1(t) [-\beta_1 S I_1 - \beta_2 (1-u) S I_2] \\ &+ \lambda_2(t) [\beta_1 S I_1 - r_1 I_1 + (p_1 - p_2) I_1 I_2 + \rho_1 I_1 R_2] \\ &+ \lambda_3(t) [\beta_2 (1-u) S I_2 - r_2 I_2 + (p_2 - p_1) I_1 I_2 + \rho_2 (1-v) I_2 R_1] \\ &+ \lambda_4(t) [r_1 I_1 - \rho_2 (1-v) I_2 R_1] \\ &+ \lambda_5(t) [r_2 I_2 - \rho_1 I_1 R_2] \end{aligned} \quad (13)$$

To find the optimal solution, we apply the Pontryagin's Maximum Principle (Pontryagin, 2018) to the Hamiltonian, and we obtain the following theorem.

Theorem 1. *Let $S^*(t), I_1^*(t), I_2^*(t), R_1^*(t)$ and $R_2^*(t)$ be optimal state solutions with associated optimal control variables $u^*(t)$ and $v^*(t)$ for the optimal control problem (11).*

Then, there exist adjoint variables $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$, $\lambda_4(t)$ and $\lambda_5(t)$ that satisfy

$$\begin{aligned}
\dot{\lambda}_1 &= \lambda_1\,(I_1\,\beta_1 - I_2\,\beta_2\,(u-1)) - I_1\,\beta_1\,\lambda_2 + I_2\,\beta_2\,\lambda_3\,(u-1)\\
\dot{\lambda}_2 &= c_1 - \lambda_4\,r_1 - \lambda_2\,(S\,\beta_1 - r_1 + R_2\,\rho_1 + I_2\,(p_1-p_2))\\
&\quad + S\,\beta_1\,\lambda_1 + R_2\,\lambda_5\,\rho_1 + I_2\,\lambda_3\,(p_1-p_2)\\
\dot{\lambda}_3 &= \lambda_3\,(r_2 + I_1\,(p_1-p_2) + S\,\beta_2\,(u-1) + R_1\,\rho_2\,(v-1))\\
&\quad - \lambda_5\,r_2 - c_2 - I_1\,\lambda_2\,(p_1-p_2) - S\,\beta_2\,\lambda_1\,(u-1) - R_1\,\lambda_4\,\rho_2\,(v-1)\\
\dot{\lambda}_4 &= I_2\,\lambda_3\,\rho_2\,(v-1) - I_2\,\lambda_4\,\rho_2\,(v-1)\\
\dot{\lambda}_5 &= I_1\,\lambda_5\,\rho_1 - I_1\,\lambda_2\,\rho_1
\end{aligned}$$

with the transversality conditions $\lambda_i(t_f) = 0$, $i = 1,2,3,4,5$. Furthermore, the optimal controls $u^*(t)$ and $v^*(t)$ are given by

$$\begin{aligned}
u^*(t) &= max\left\{min\left\{\frac{\lambda_3-\lambda_1}{K_1} I_2\,S\,\beta_2, 1\right\}, 0\right\}\\
v^*(t) &= max\left\{min\left\{\frac{\lambda_3-\lambda_4}{K_2}\,\rho_2 I_2\,R_1, 1\right\}, 0\right\}
\end{aligned}$$

Proof. (Pontryagin, 2018) To determine the adjoint equations and the transversality conditions, we use the Hamiltonian $\mathcal{H}$ defined by (13). From setting $I(t) = I^*(t)$, $A(t) = A^*(t)$ and $D(t) = D^*(t)$, and differentiating $\mathcal{H}$ with respect to $I(t)$, $A(t)$ and $D(t)$, we obtain

$$\begin{aligned}
\dot{\lambda}_1 &= -\frac{\partial \mathcal{H}}{\partial S}\\
&= \lambda_1\,(I_1\,\beta_1 - I_2\,\beta_2\,(u-1)) - I_1\,\beta_1\,\lambda_2 + I_2\,\beta_2\,\lambda_3\,(u-1)\\
\dot{\lambda}_2 &= -\frac{\partial \mathcal{H}}{\partial I_1}\\
&= c_1 - \lambda_4\,r_1 - \lambda_2\,(S\,\beta_1 - r_1 + R_2\,\rho_1 + I_2\,(p_1-p_2))\\
&\quad + S\,\beta_1\,\lambda_1 + R_2\,\lambda_5\,\rho_1 + I_2\,\lambda_3\,(p_1-p_2)\\
\dot{\lambda}_3 &= -\frac{\partial \mathcal{H}}{\partial I_2}\\
&= \lambda_3\,(r_2 + I_1\,(p_1-p_2) + S\,\beta_2\,(u-1) + R_1\,\rho_2\,(v-1))\\
&\quad - \lambda_5\,r_2 - c_1 - I_1\,\lambda_2\,(p_1-p_2) - S\,\beta_2\,\lambda_1\,(u-1) - R_1\,\lambda_4\,\rho_2\,(v-1)\\
\dot{\lambda}_4 &= -\frac{\partial \mathcal{H}}{\partial R_1}\\
&= I_2\,\lambda_3\,\rho_2\,(v-1) - I_2\,\lambda_4\,\rho_2\,(v-1)\\
\dot{\lambda}_5 &= -\frac{\partial \mathcal{H}}{\partial R_2}\\
&= I_1\,\lambda_5\,\rho_1 - I_1\,\lambda_2\,\rho_1
\end{aligned}$$

By the optimality conditions, we have

$$\frac{\partial \mathcal{H}}{\partial u} = K_1\,u + I_2\,S\,\beta_2\,\lambda_1 - I_2\,S\,\beta_2\,\lambda_3 \quad = \quad 0$$

Table 2: Parameters values

Parameter	$S(0)$	$I_1(0)$	$I_2(0)$	$R_1(0)$	$R_2(0)$	β_1	β_2	p_1	p_2	ρ_1	ρ_2	r_1	r_2
Value	300	1	1	1	1	0.00091	0.0005816	0.0003	0.0004	0.0002	0.0003	0.01	0.001

then

$$u(t) = \frac{\lambda_3 - \lambda_1}{K_1} I_2\, S\, \beta_2$$

and from

$$\frac{\partial \mathcal{H}}{\partial v} = K_2\, v - I_2\, R_1\, \lambda_3\, \rho_2 + I_2\, R_1\, \lambda_4\, \rho_2 \quad = \quad 0$$

we have

$$v(t) = \frac{\lambda_3 - \lambda_4}{K_2}\, \rho_2 I_2\, R_1$$

As our controls are bounded below by 0 and above by 1, thus we have

$$\begin{aligned} u^*(t) &= max\left\{min\left\{\frac{\lambda_3 - \lambda_1}{K_1} I_2\, S\, \beta_2, 1\right\}, 0\right\} \\ v^*(t) &= max\left\{min\left\{\frac{\lambda_3 - \lambda_4}{K_2}\, \rho_2 I_2\, R_1, 1\right\}, 0\right\} \end{aligned}$$

□

4 Numerical simulation

We provide numerical simulations of our optimal system that was developed based on the previous mathematical model. To conduct these simulations, we implemented a MATLAB^{TM} code and utilized a range of data sets to test our results. The optimization problems were solved using an iterative method, employing a progressive-regressive Runge–Kutta fourth-order scheme. This numerical approach is referred to as a forward-backward sweep method, where the state system is initially solved forward in time using an initial guess, followed by solving the adjoint system backward in time. By utilizing this numerical procedure, we are able to effectively analyze the optimal control strategy for information spread and evaluate the effectiveness of our mathematical model. First, starting with an initial guess for the adjoint variables λ_1, λ_2, λ_3, λ_4, and λ_5, we solve the state equations by a forward Runge–Kutta fourth-order procedure in time. Then, those state values are used to solve the adjoint equations by a backward Runge–Kutta fourth order procedure because of the transversality conditions (Zakary et al., 2016, 2017, 2020; Jung et al., 2002; Lenhart & Workman, 2007). Afterwards, we updated the optimal control values using the values of state and co-state variables obtained in the previous steps. Finally, we execute the previous steps until a tolerance criterion is reached. In order to show the importance of our work, we consider here an example of data given in Table 2.

Without controls

Fig.1 depicts the state variables S, I_1, I_2, R_1, and R_2 of the model (1)-(5) when there is no control intervention. It can be seen that from the beginning of the simulation, the number of people reached by information 1, I_1, increases significantly, then begins to decrease continuously until the end of the simulation. While the number of people reached by the misinformation (Information 2) I_2 increases rapidly, and they continue to increase until the end of the simulation.

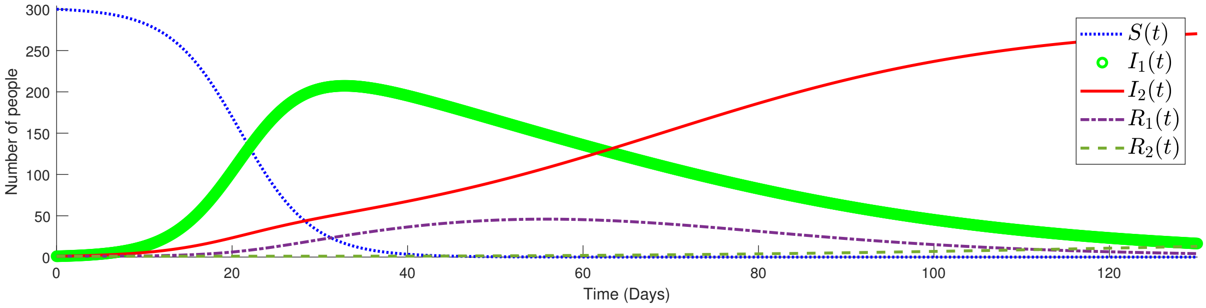

Figure 1: The model without controls

The number of Indifferent people decreases quickly in about 35 days, from 300 to about 10, to begin to decrease slightly.

The number of R_1 rises about 50 persons and begin decreasing slightly, while the number of R_2 people keeps small values around 10 persons at the end of the simulation.

With controls

Strategy 1: Combination of the Fact-checking and Media literacy programs

Fig.2 (a) depicts the state variables S, I_1, I_2, R_1, and R_2 of the model (6)-(10) when the both control interventions are used. It can be seen that from the beginning of the simulation, the number of people reached by information 1, I_1, increases significantly to reach more than 260 persons, then begins to decrease slightly until the end of the simulation. While the number of people reached by the misinformation (Information 2) I_2 increases slowly, and they continue to increase until the end of the simulation under 100 persons compared to more than 290 persons when there is no control. The number of Indifferent people decreases quickly in about 30 days, from 300 to about 10, to begin to decrease slightly.

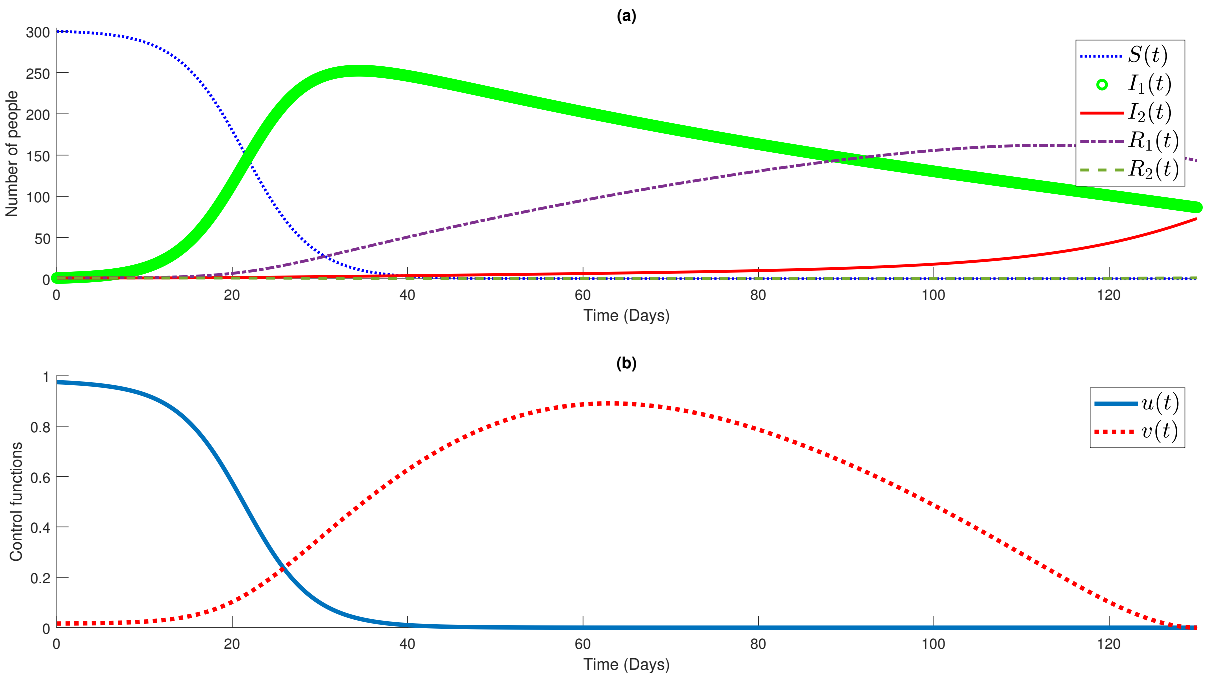

Figure 2: The model with controls

The number of R_1 rises about 170 persons in about 115 days and begin decreasing slightly, while the number of R_2 people keeps small values around 5 persons from the beginning to the end of the simulation.

Fig.2 (b) depicts the control function u and v used in this strategy of control.

In addition to their impact on the spread of misinformation, the control variables u and v can also affect the accuracy of information transmission. Media literacy programs can help individuals identify and interpret reliable sources of information, which can increase the accuracy of information dissemination. Fact-checking can reduce the propagation of inaccurate information, but it can also lead to the propagation of overly cautious and conservative information. This is because fact-checkers may err on the side of caution and reject information that is uncertain but still valuable. which justifies the higher number of R_1 in Fig.2 (a) compared to the small number in the Fig.1. Therefore, there is a trade-off between accuracy and speed of information transmission that must be considered when evaluating the impact of these controls. Additionally, the impact of these control variables may vary depending on the nature of the information being spread and the characteristics of the population receiving it. For example, media literacy programs may be more effective for individuals with high levels of education, while fact-checking may be more effective for individuals with a greater distrust of information sources.

Strategy 2: Using only the Media literacy programs

Fig.3 (a) depicts the state variables S, I_1, I_2, R_1, and R_2 of the model (6)-(10) when only the control u is used. It can be seen that from the beginning of the simulation, the number of people reached by information 1, I_1, increases significantly to reach more than 250 persons, then begins to decrease slightly until the end of the simulation. While the number of people reached by the misinformation (Information 2) I_2 increases slowly, and they continue to increase until the end of the simulation to reach about 200 persons compared to more than 290 persons when there is no control. The number of Indifferent people decreases quickly in about 30 days, from 300 to about 10, to begin to decrease slightly.

The number of R_1 rises about 140 persons in about 100 days and begin decreasing slightly, while the number of R_2 people keeps small values around 5 persons from the beginning to the end of the simulation.

Fig.3 (b) depicts the control function u and v used in this simulation.

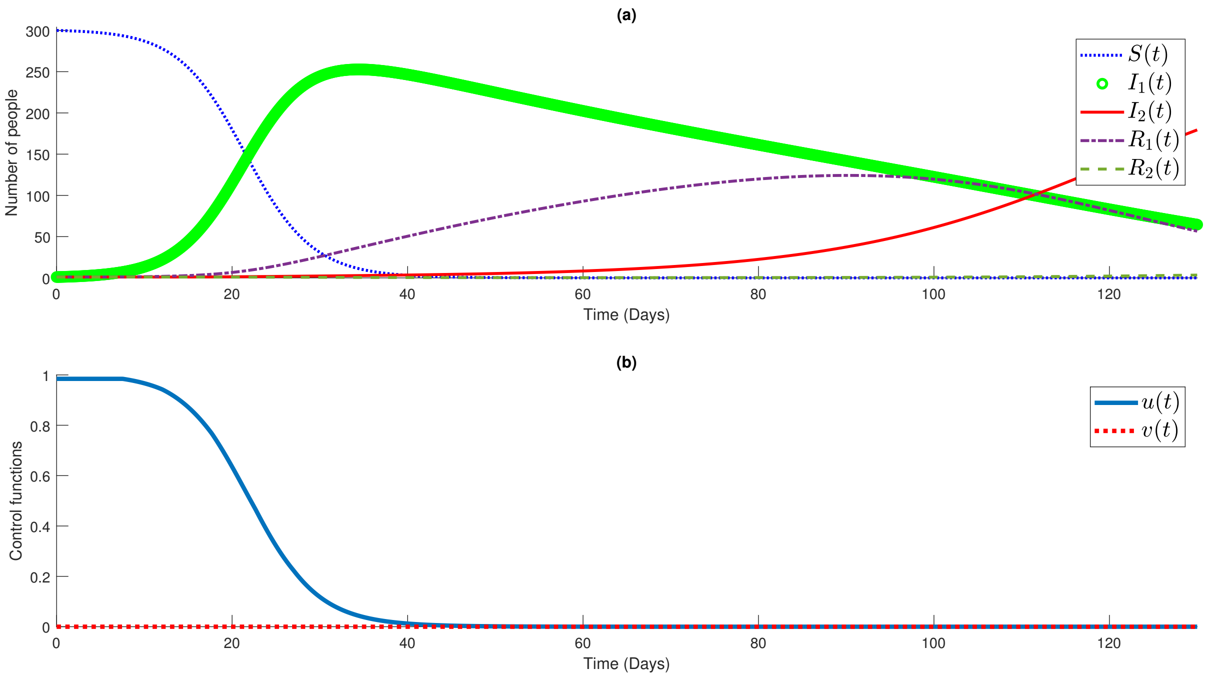

Figure 3: The model with only u control

By increasing the public's ability to critically analyze and evaluate the veracity of information, individuals may be better equipped to identify and discard false or misleading information. This can lead to a more informed and educated population, and ultimately contribute to the dissemination of accurate and reliable information. Additionally, media literacy programs can increase the public's awareness of the potential harms associated with the spread of rumors and misinformation, and promote a more responsible and ethical use of information. However, the effectiveness of media literacy programs may depend on various factors, such as the specific design and implementation of the program, the target audience, and the prevailing cultural and social norms.

Strategy 2: Using only the Fact-checking control

Fig.4 (a) depicts the state variables S, I_1, I_2, R_1, and R_2 of the model (6)-(10) when only the control v is used. It can be seen that from the beginning of the simulation, the number of people reached by information 1, I_1, increases significantly to reach about 210 persons, then begins to decrease slightly until the end of the simulation. While the number of people reached by the misinformation (Information 2) I_2 increases quickly compared to strategy 2 and 1, and they continue to increase until the end of the simulation to reach about 250 persons compared to more than 290 persons when there is no control. The number of Indifferent people decreases quickly in about 30 days, from 300 to about 10, to begin to decrease slightly.

The number of R_1 rises about 110 persons in about 90 days and begin decreasing slightly, while the number of R_2 people keeps small values around 15 persons at the end of the simulation.

Fig.4 (b) depicts the control function u and v used in this simulation.

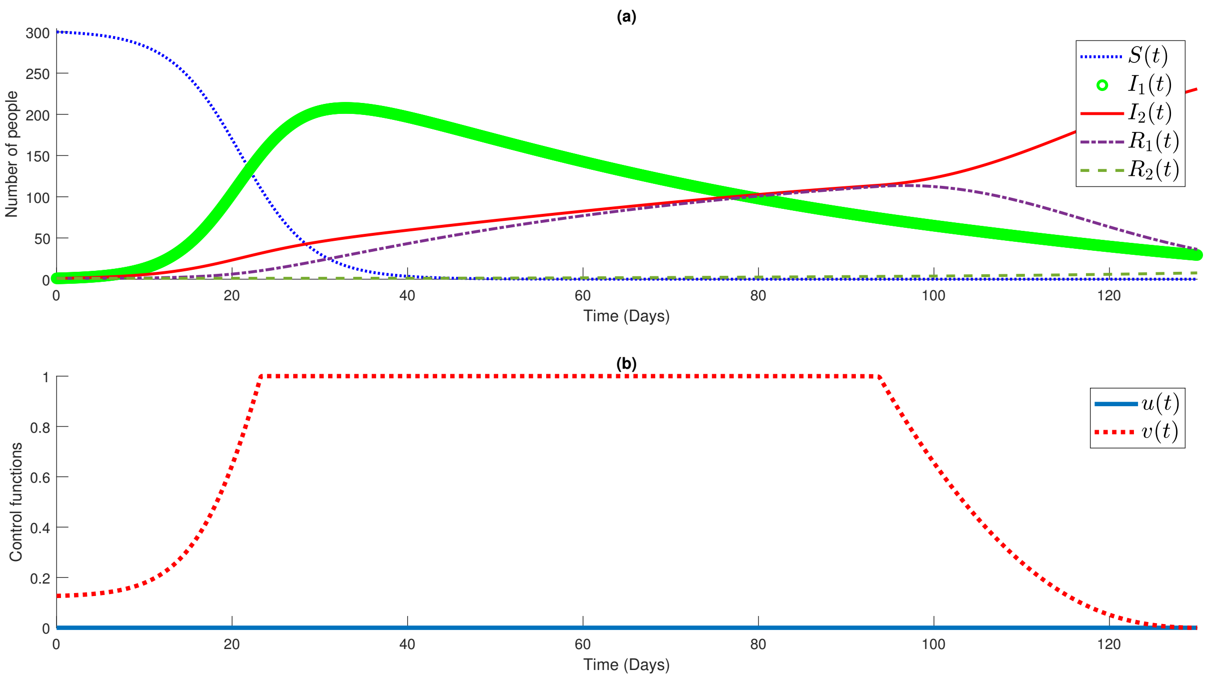

Figure 4: The model with only v control

The use of fact-checking can help mitigate the spread of misinformation and rumors by providing credible and accurate information that can counter false narratives.

On can see that the Fact-checking control can help stop the spread of misinformation by providing alternative, credible sources of information. It can help break the chain of misinformation and reduce its overall impact. By providing a credible source of information, fact-checking can increase public trust in information and reduce the influence of false narratives. It can also provide individuals with accurate information, allowing them to make more informed decisions based on reliable sources.

It is important to note that the effectiveness of fact-checking can depend on various factors, such as the credibility of the fact-checkers, the reach of the fact-checking information, and the willingness of individuals to engage with it. Additionally, there may be challenges in implementing fact-checking at scale, such as the need for a large number of fact-checkers and the resources required to fact-check a large volume of information.

Comparison

Our analysis shows that the use of both control measures leads to a significantly higher number of people reached by information 1 compared to the other strategies. The results are illustrated in Fig.5 (a), where the curves represent the number of people reached by information 1 under different control scenarios. The first curve represents the scenario with no control, which results in a gradual decline in the number of people I_1 reached by information 1 over time. The second curve labeled "With only v" corresponds to the scenario with only fact-checking control (**Strategy 3**), which initially leads to an increase in the number of people reached by this information. The third curve labeled "With only u" represents the scenario with only media literacy programs (**Strategy 2**), which leads to a higher increase in the number of people I_1 reached by information 1. The fourth curve labeled "With all controls" represents the scenario where both controls are implemented (**Strategy 1**), resulting in a steady increase in the number of people reached by this information over time. These results highlight the importance of implementing a combination of control measures in promoting the dissemination of accurate information.

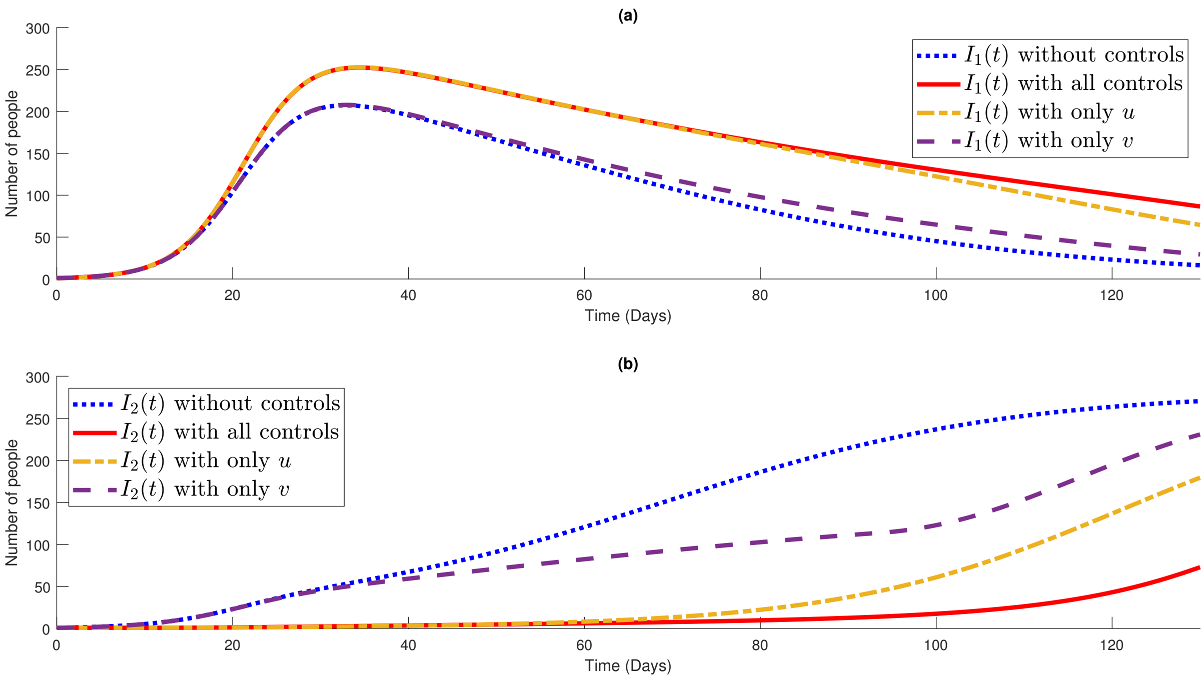

Figure 5: Comparison of the three strategies: I_1 and I_2

Fig.5 (b) displays the results of our simulations comparing the effect of the three different control strategies on the spread of information 2. The four curves represent the number of people reached by information 2, I_2, over time, with the x-axis showing time in days and the y-axis indicating the number of people reached. The first curve, labeled "Without controls" represents the spread of information 2 without any intervention. As expected, the curve shows a rapid increase in the number of people reached by information 2, followed by a plateau as the majority of the population becomes aware of the information.

The second curve, labeled "With only v," represents the spread of information 2 when using only a fact-checking control v (**Strategy 3**). This curve shows a significant decrease in the number of people reached by information 2 compared to the "No control" curve, indicating that the use of fact-checking can be an effective strategy to mitigate the spread of false information.

The third curve, labeled "With only u," represents the spread of information 2 when using media literacy programs control u (**Strategy 2**). This curve shows a similar decrease in the number of people reached by this information compared to the "No control" curve. This result suggests that media literacy programs can also be effective in promoting the dissemination of accurate information and reducing the spread of misinformation.

Finally, the fourth curve, labeled "With all controls," represents the spread of information 2 when using both fact-checking and media literacy programs (**Strategy 1**). This curve shows the most significant decrease in the number of people reached by this information compared to the "No control" curve and suggests that combining multiple strategies can be an effective approach to mitigate the spread of rumors and misinformation. Overall, the results of this figure highlight the importance of implementing effective control strategies to promote accurate information dissemination and reduce the spread of false rumors.

Fig.6 (a) shows the results of our simulations of the number of people who lose interest in information 1 over time, comparing three different control strategies with a no-control scenario. The curves depict the number of people who lose interest in information 1, R_1, as a function of time, with the x-axis representing time in days and the y-axis representing the number of people.

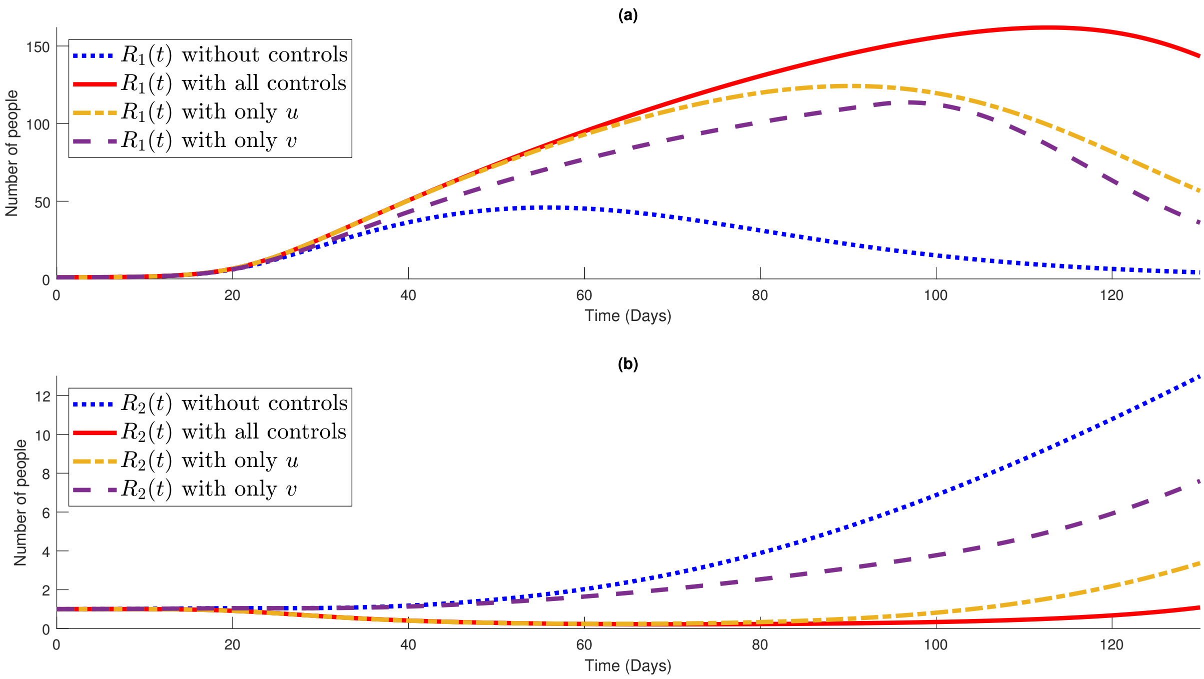

Figure 6: Comparison of the three strategies: R_1 and R_2

The numerical simulations suggest that the combination of media literacy programs and fact-checking is an effective strategy for mitigating the spread of inaccurate information. The results indicate that the number of people reached by accurate information is larger when both controls are implemented. However, this also suggests that the number of people who lose interest in accurate information is higher when both controls are used. This result is logical because, with increased media literacy, individuals are better equipped to think critically about the information they encounter. As a result, they may be more likely to recognize misinformation and refrain from sharing it. In addition, fact-checking efforts can help prevent the spread of rumors by identifying and correcting inaccurate information. While the increase in the number of people losing interest in accurate information is a potential concern, it is important to consider the overall impact of the controls on the spread of misinformation.

In (b), as expected, the number of people who lose interest in information 2 increases over time in the no-control scenario due to the bigger number of people reached by this information. However, when either control strategy is implemented, the rate at which people lose interest in the information decreases, with the greatest reduction seen in the scenario where both control strategies are used.

5 Statistical analysis

In this section, we present a statistical analysis of the number of people reached by information 2 using the different strategies of control. We created a table (Table 3) that displays the mean, standard deviation, and median for each control strategy. Our result shows that the combination of media literacy programs and fact-checking had the smallest mean, indicating that this strategy was the most effective in reducing the spread of information 2. This result is further supported by the fact that the standard deviation for this strategy was also smaller than the other strategies. The median for this strategy was also smaller than the other strategies, indicating that the majority of the data falls within a narrower range. The fact-checking strategy alone had the largest mean, indicating that it was the least effective in reducing the spread of information 2. The media literacy programs alone had a larger mean than the combined strategy, indicating that it was not as effective in reducing the spread of information 2 as the combined strategy. Overall, these results suggest that the combination of media literacy programs and fact-checking is the most effective strategy for reducing the spread of information 2, and should be prioritized in efforts to promote the dissemination of accurate information.

Table 3: Mean, Standard deviation and the median of each strategy of control

	Mean	**Std**	**Median**
Strategy 1	13.2924	15.8402	7.1043
Strategy 2	36.5847	49.0945	10.5956
Strategy 3	90.7782	58.8445	87.9814

The distribution of data is an important aspect to consider when analyzing results. In Fig.7, histograms are presented to visualize the distribution of the number of people reached by information 2 for each control strategy. It is clear from the histograms that none of the distributions follow a normal distribution and the p-values of the Shapiro-Wilk and the Lilliefors tests confirm it (since we reject the null hypothesis of normality for all strategies at the 5% significance level). This can have implications for the use of certain statistical tests, and it highlights the need to carefully select appropriate methods for data analysis.

After showing that the histograms in Figure 3 do not seem to follow a normal distribution, we concluded that the normality assumption is violated for all strategies. Therefore, we performed the Wilcoxon test, a non-parametric statistical test that can be used to compare two groups of data without making any assumptions about their underlying distributions. This test is particularly useful when the data do not meet the assumptions required for a parametric test such as the t-test. By performing the Wilcoxon test, we can determine whether the differences between the groups are statistically significant, even if the data are not normally distributed. This allows us to draw more accurate conclusions about the effectiveness of the different strategies and identify the most effective approach to control the spread of information.

Fig.8 shows the means of three different strategies for the number of people reached by information 2. The bars in the figure represent the mean values for each strategy. As can be seen, Strategy 3 has the highest mean, followed by Strategy 2 and then Strategy 1.

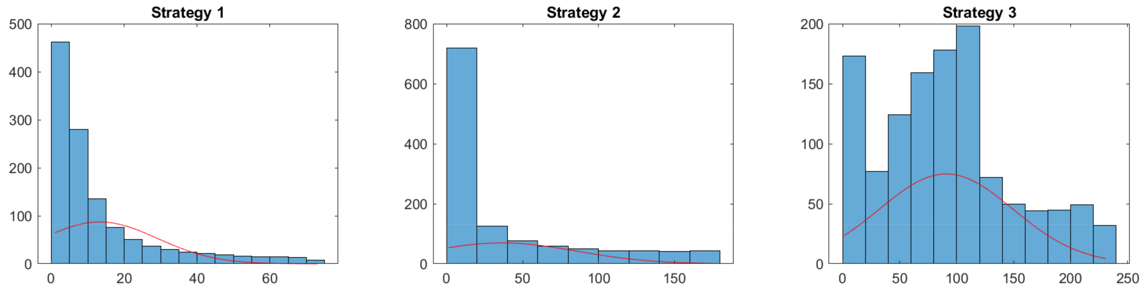

Figure 7: Normality test

The box plot in Fig.8 shows the distribution of the number of people reached by information 2, I_2, for each control strategy. The plot indicates that the median and interquartile range of the number of people reached by information 2 are lowest for strategy 1, which is the combination of media literacy programs and fact-checking. The whiskers of the box plot show the range of the data, with any outliers beyond the whiskers plotted as individual points. The box plot clearly demonstrates that strategy 1 is the best approach to decrease the number of people reached by this information. The other strategies have a wider spread of data and higher medians, indicating that they are less effective in limiting the spread of this information. Overall, the box plot provides a visual representation of the statistical analysis and supports the conclusion that strategy 1 is the most effective approach for controlling the spread of misinformation.

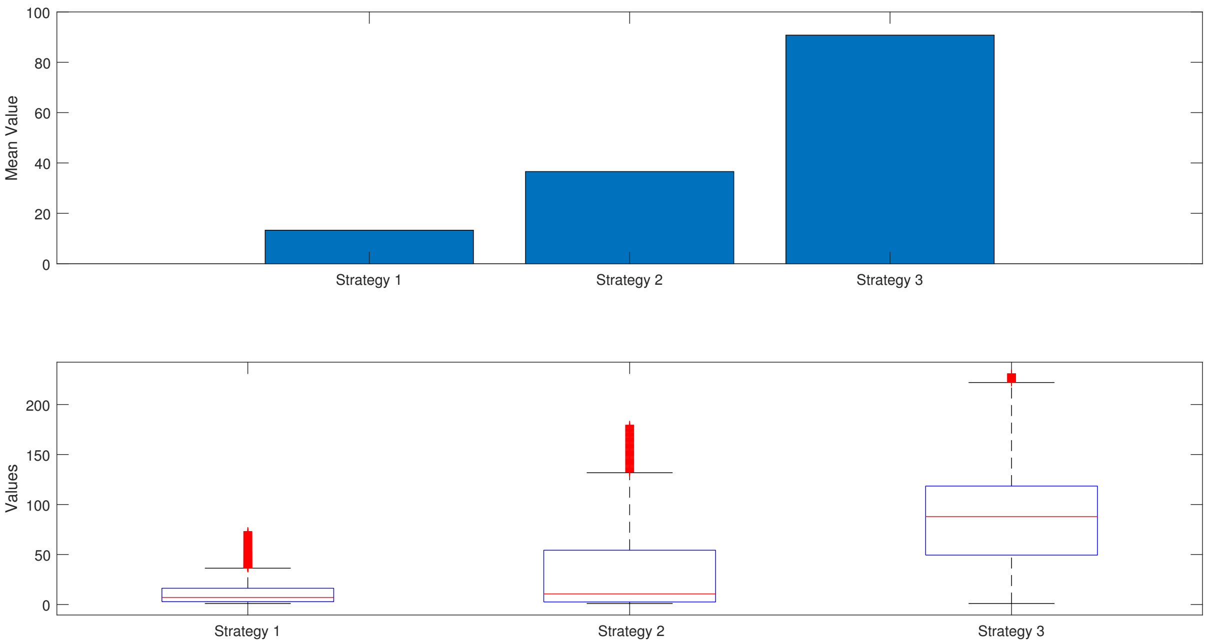

Figure 8: Means and box-plots

In order to confirm the result of the box plot, we performed a Wilcoxon test to compare the three strategies. The test was done for each pair of strategies and the results showed that the difference between the means was statistically significant (p ¡ 0.05) for all pairs. The Wilcoxon test confirmed that strategy 1 was the best strategy for decreasing the number of people reached by information 2, as indicated by the box plot.

Furthermore, Strategy 2 is significantly better than Strategy 3 in this regard. These results confirm the trends observed in the box plots, with Strategy 1 showing the lowest median number of people reached by information 2, followed by Strategy 2 and then Strategy 3. Therefore, our analysis suggests that a combination of media literacy programs and fact-checking, as implemented in Strategy 1, is the most effective in reducing the spread of inaccurate information, and should be a priority for policy makers and organizations seeking to combat the harmful effects of false information.

6 Conclusion

The aim of this study was to investigate the effectiveness of different strategies in controlling the spread of two types of information within a population. We presented a mathematical model to study the dynamics of information spread and the impact of information on other information. Our numerical simulations showed that the combination of media literacy programs and fact-checking is the best strategy to increase the number of people reached by accurate information while decreasing the number of people reached by inaccurate information. Statistical analysis was performed, including a Wilcoxon test, to compare the means and distributions of the number of people reached by each type of information under the three different strategies. The results showed that strategy 1 is the most effective in decreasing the number of people reached by inaccurate information, while strategy 2 is more effective in increasing the number of people reached by accurate information. The histograms and box plots indicated that none of the strategies followed a normal distribution, and the Wilcoxon test confirmed the results of the box plots. These findings have important implications for efforts to control the spread of misinformation and promote the dissemination of accurate information within a population.

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